Subjective Expected Utility Theory and Prospect Theory: Critical Comparison and Practical Application

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Abstract: This paper reviews the development process of Subjective Expected utility Theory and Prospect Theory, including the core viewpoints and formula models of the two theories. Examples are given to illustrate the contributions of the two theories in the study of human decision-making.

Keywords： Subjective Expected; Utility Theory Prospect ;Theory Risk Decision

1. Subjective Expected Utility Theory

1.1 SEUT concept and development

Subjective expected utility theory (SEUT) is a part of the expected utility theory system, which derives from the Von Neumann axiom system. Von Neumann established the expected utility maximisation principle based on the interpretation of the St. Petersburg paradox[1]. That is, if a random variable X is assessed with probability Pi xi, I = 1,2,... ,n, then a person's utility u(xi) when he is certain to acquire Xi is equal to: U(X) = E[u(X)] = P1u(x1) + P2u(x2) +... + Pnu (xn).

In other words, the decision-maker' decision is a trade-off between utility u and probability P. For example, a rational man is afraid to pursue a beautiful woman. The possible reason is that he thinks that although he is attracted to this beautiful woman, which is called utility U, his probability of success in this pursuit is low.

The SEU theory includes subjective and personal factors, then summarises the subjective probability of the possible occurrence of something. Based on the expected utility theory (EUT), Savage[2] refined subjective expected utility theory (SEUT).

In the subjective probability model, the total of the subjective probabilities of all possible outcomes equals 1: $\sum_{i=1}^{n} \pi i = 1$. In other words, when the safety factor is directly substituted for the objective probability, SEUT uses the subjective probability model to solve the problem that the total probability sum of the objective probability is not 1.

The subjective probabilityis substituted for the objective probability Pi in the expected utility model, and the subjective expected utility model is obtained. One critical contribution of the subjective expected utility theory is how to distinguish uncertainty from the risk attitude of decision-makers.

1.2 Criticism of SEUT

Due so many insurmountable obstacles, using the SEU theory in its original form to make practical judgments is unfeasible[3]. The EU and SEU theories describe the decision-making behaviour of "rational man" under risk settings. However, man is not a purely rational being. His decisions are influenced by complications in his thought processes. As a result, the descriptive validity of the EU theory on human risk decisions has long been questioned.

1.3 Allais Paradox

If the connection between the subjective anticipated utility values of two acts is UA＞UB, the sure-thing principle in SEU argues that the inequality will still hold when common values are added to both sides of this inequality. In 1952, French economist Maurice Allais posed a challenge to this certainty premise, commonly known as the Allais Paradox[4]. The contradiction is illustrated as follows:

The first experiments

A:Sure you get $1,000,000

B:10% chance: winning $5,000,000; 89% probability: winning $1,000,000;1% chance :winning nothing.

The second experiments

C:11% chance: winning $1,000,000;89% probability: getting nothing

D:10% chance: winning $5,000,000;90% of the time: you get nothing

The preference connection of ABCD should be totally consistent in accordance with the sure-thing principle of the subjective anticipated value theory.

Simply put, if participants chose A or B in the first set of trials, the utility inequality dictates that they would choose C or D in the second set of experiments. However, trial results show that most individuals chose A and D, which contradicts the subjective anticipated utility hypothesis.

Although Savage was aware that his first choice violated the expected utility of independence and immediately correct choice, he believed that rational people would modify their own preference relations. In other words, when rational persons are aware of their preference after violating the sure-thing principle, they will revise, and their final choice would conform to the sure thing principle.

Unfortunately, Slovic and Tversky's experiment found that, when the statement got clearer, the number of people who turned against expected utility was equal to the number of people who turned for expected utility[5].

2. Prospect Theory

2.1 Prospect theory concept and development

To explain the Allais Paradox , Kahneman and Tversky[6] proposed the prospect theory. According to Kahneman, the anticipated utility theory is based on the completeness axiom of preferences and the transitivity axiom of preferences. The expected utility theory is no longer viable if these two axioms are not fulfilled. Consequently, the utility function of predicted utility theory is replaced by the value function in prospect theory.

In prospect theory, a reference point is a type of assessment standard that is subjectively established by individuals and varies with different evaluation topics, environments, and times. The decision-making process in prospect theory includes two stages: editing and valuation. The editing stage restructures the alternatives, analyses the advantages and probabilities associated with the offered expectations, and eases the decision-making process. The valuation stage follows after editing. Here, the decision-maker values each revised Prospect and chooses the best one. A weighted linear sum of the values and weights is used to calculate the overall value of the Prospect. The value function and the decision weight function are discussed in a later section.

According to prospect theory, people have the following risk attitudes: (a) risk aversion toward high probability gain and risk seeking toward high probability loss, and (b) a preference for risk in relation to low probability gains and an aversion to low probability losses. These attitudes give rise to the key prospect theory model:where

the value function V (xi) and the weight function W(pi) work together to determine the expected value V. The value function is expressed as follows:

$$v(x) = \begin{cases} x^{\beta}, x \geq 0 \\ -\theta(-x)^{\beta}, x \leq 0 \end{cases}, \beta \in [0,1], \theta > 1$$

According to Camerer and Ho (1994), the weight function is expressed as follows:

$$w^{+}(p) = w^{-}(p) = w(p) = \frac{p^{\varepsilon}}{\left[p^{\varepsilon} + (1-p)^{\varepsilon}\right]^{1/\varepsilon}}, \varepsilon \in [0,1]$$

In the value function, x represents the relative gain or loss to the reference point and θ represents the risk aversion coefficient. The weight function includes two parameters:for gains and w^{-}(p for losses.

3. Difference between prospect theory and subjective expected utility theory

3.1 Differences in convexity

Because the utility function in subjective expected utility theory is always convex, people are always risk-averse. In perspective theory, the value function is s-shaped, concave in the direction of gains but convex in the direction of losses, with declining sensitivity in both directions.

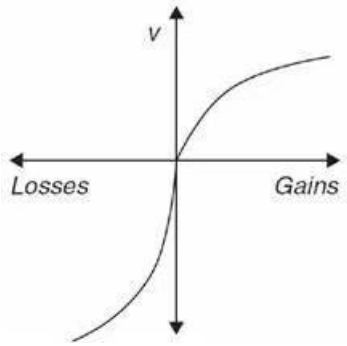

Figure 1 Value function

The slope of the loss region is bigger than that of the gain area. Moreover, according to the value function graph, the function of the loss section is steeper than that of the gain section. As shown in this pattern, the negative value of loss is larger than the positive utility of gain by the same amount. Simply put, people experience more psychological alterations as a result of loss than they do as a result of gain.

For example, we are playing a coin toss. I'll give you $1,100 if you win, and I'll give you $1,000 if you lose. From the perspective of expected value theory, this is a game that is entirely in your favour. In practice, however, it is likely that most people will not play the game because they know that there is a 0.5 chance of losing. This is the prospect theory of irrational behaviour loss aversion. We often see this in the marketing strategy used in many stores, where the store tells the consumer: “If you don't order today, you will lose the offer.” Many merchants exploit people's fear of losing to drive transactions.

3.2 Nonlinearity of the decision weight function

Unlike the use of objective and subjective probabilities in EU and SEU, the decision weight function is a nonlinear function rather than subjective or objective probability. Moreover, the function is monotonically increasing.

In other words, prospect theory states that, during a decision-making process, individuals would use their prior experience and subjective considerations to make subjective changes to the objective likelihood of risk occurrences.

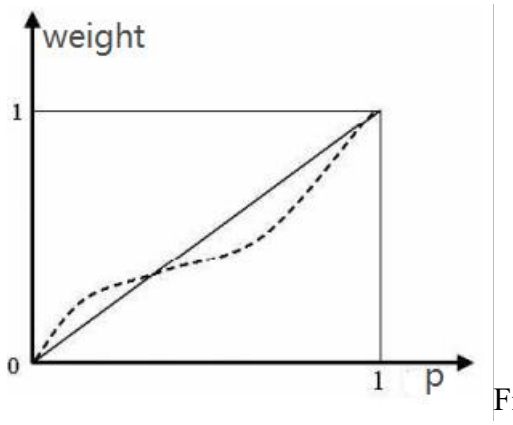

Figure 2 Decision Weight Function

In the preceding figure, the dotted line indicates the weight function, whilst the solid line indicates the probability function. The figure shows that, in the low probability stage, the weight function is greater than the probability function. On the other hand, in the high probability stage, the weight function is less than the probability function.

In other words, in the decision weight function, humans tend to overestimate

low-probability events and underestimate high-probability events. At the same time, they tend to be indifferent to changes in probability in the intermediate stage. This idea explains why humans pay excessive attention to low-probability occurrences whilst ignoring ordinary happenings. Moreover, this theory can explain the option smile phenomenon, where investors' high decision weight on low-probability events lead to the overvaluation of stock options.

4. Conclusions

This article presents the key concepts and compares the differences between the subjective expectation effect theory and the prospect theory.

The prospect theory defines short-term behaviour, whereas the subjective expected utility theory addresses long-term behaviour. Consequently, the subjective expectation effect hypothesis and the prospect theory are consistent[7].

However, compared to anticipated value theory, prospect theory is closer to reality, better represents social and economic concerns, and thus more useful in the context of behavioural economics.

In the classic research approach of expected utility theory, human subjectivity is ignored. However, humans behaviour are often illogical and show a variety of cognitive biases. For instance, according to behavioural economics, human conduct is selfish and conditioned by societal ideals to act in ways that do not maximise advantages.

In research practise, both the subjective expectation effect theory and the prospect theory are widely applied. For example, the value of life theory, which is derived from the expectation effect theory, can help enterprises balance employee compensation and risk. Moreover, prospect theory can help analyse human behaviour in fields such as financial investment, marketing, and psychology.

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